

Experimental approaches to the difference in the Casimir force through the varying optical properties of boundary surface

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Abstract

We propose two novel experiments on the measurement of the Casimir force acting between a gold coated sphere and semiconductor plates with markedly different charge carrier densities. In the first of these experiments a patterned Si plate is used which consists of two sections of different dopant densities and oscillates in the horizontal direction below a sphere. The measurement scheme in this experiment is differential, i.e., allows the direct high-precision measurement of the difference of the Casimir forces between the sphere and sections of the patterned plate or the difference of the equivalent pressures between Au and patterned parallel plates with static and dynamic techniques, respectively. The second experiment proposes to measure the Casimir force between the same sphere and a VO₂ film which undergoes the insulator-metal phase transition with the increase of temperature. We report the present status of the interferometer based variable temperature apparatus developed to perform both experiments and present the first results on the calibration and sensitivity. The magnitudes of the Casimir forces and pressures in the experimental configurations are calculated using different theoretical approaches to the description of optical and conductivity properties of semiconductors at low frequencies proposed in the literature. It is shown that the suggested experiments will aid in the resolution of theoretical problems arising in the application of the Lifshitz theory at nonzero temperature to real materials. They will also open new opportunities in nanotechnology.

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I. INTRODUCTION

The Casimir effect [1] implies that there is a force acting between closely spaced electrically neutral bodies following from the zero-point oscillations of the electromagnetic field. The Casimir force can be viewed as an extension of the van der Waals force to large separations where the retardation effects come into play. Within a decade of Casimir's work, Lifshitz and collaborators [2, 3] introduced the role of optical properties of the material into the van der Waals and Casimir force. In the last few years, the advances following from both fundamental physics and nanotechnology have motivated careful experimental and theoretical investigations of the Casimir effect. The first modern experiments were made with metal test bodies in a sphere-plate configuration, and their results are summarized in Ref. [4]. In subsequent experiments the lateral Casimir force between corrugated surfaces [5] and the pressure in the original Casimir configuration [6] have been demonstrated. Later experiments [7, 8, 9] have brought the most precise determination of the Casimir pressure between two metal plates. The rapid theoretical progress has raised fundamental questions on our understanding of the Casimir force between real metals at nonzero temperature. Specifically, the role of conductivity processes and the related optical properties of metals at quasi-static frequencies has become the subject of discussions [10, 11, 12, 13, 14, 15, 16, 17].

One of the most important applications of the Casimir effect is the design, fabrication and function of micro- and nanoelectromechanical systems such as micromirrors, microresonators, nanotweezers and nanoscale actuators [18, 19, 20, 21, 22]. The separations between the adjacent surfaces in such devices are rapidly falling below a micrometer, i.e., to a region where the Casimir force becomes comparable with typical electrostatic forces. It is important that investigations of the Casimir force be done in semiconductors as they are the material of choice for the fabrications of optomechanical, micro- and nanoelectromechanical systems. While the role of conductivity and optical properties of materials can be checked in metals, semiconductors offer better control of the related parameters (charge carrier density, defect density, size etc.) and will provide an exhaustive check of the various models.

Reference [23] pioneered the measurement of the Casimir force acting between a semiconductor surface, single crystal Si wafer, and a gold coated sphere. The experimental data obtained for a wafer with the concentration of charge carriers $\approx 3 \times 10^{19} \text{ cm}^{-3}$ were compared with the Lifshitz theory at zero temperature and good agreement was found at a

95% confidence level. At the same time, the theory describing a “dielectric” Si plate with a concentration $\sim 5 \times 10^{12} \text{ cm}^{-3}$ was excluded by experiment at 70% confidence. This allows one to conclude that the Casimir force is sensitive to the conductivity properties of semiconductors. This conclusion has found direct experimental confirmation in Ref. [24] where the Casimir forces between a gold coated sphere and two different Si wafers with the concentrations of charge carriers $\approx 3.2 \times 10^{20} \text{ cm}^{-3}$ and $1.2 \times 10^{16} \text{ cm}^{-3}$ have been measured. The difference of the measured forces for the two conductivities was found to be in good agreement with the corresponding difference of the theoretical results computed at zero temperature (note that the sensitivity of force measurements in Refs. [23, 24] was not sufficient to detect the thermal corrections predicted in Refs. [10, 11, 12, 13, 14, 15, 16, 17]).

In the most precise experiment on the Casimir force between a metal and a semiconductor [25], the density of charge carriers in a Si membrane of $4 \mu\text{m}$ thickness is changed from $5 \times 10^{14} \text{ cm}^{-3}$ to $2 \times 10^{19} \text{ cm}^{-3}$ through the absorption of photons from a laser pulse. This is a differential experiment where only the difference of the Casimir forces in the presence and in the absence of a laser pulse was measured. This decreases the experimental error to a fraction of a 1 pN and allows one to check the role of conductivity processes in semiconductors at the laboratory temperature $T = 300 \text{ K}$. The experimental data for the difference Casimir force as a function of separation were compared with the Lifshitz theory and the outcome was somewhat puzzling. The data were found to be in excellent agreement with the theoretical difference force computed at $T = 300 \text{ K}$ under the assumption that in the absence of laser light Si possesses a finite static dielectric permittivity. By contrast, if theory takes into account the dc conductivity of Si in the absence of laser light, it is excluded by the data at a 95% confidence level. This is somewhat analogous to the above-mentioned problems for two real metals where the inclusion of the actual conductivity processes at low frequencies also leads to disagreement with experiment [7, 8, 9]. The fundamental questions on the role of scattering processes and conductivity at low frequencies in the Casimir force have to be clarified for further progress in the field and this calls for new precise experiments.

In this paper we propose two experiments on the Casimir force between a metal sphere and semiconductor plate which can shed light on the applicability of the Lifshitz theory at nonzero temperature to real materials. In the first of these experiments, the patterned Si plate with two sections of different dopant densities is oscillated in the horizontal direction below the Au coated sphere. As a result, the sphere is subject to the difference Casimir force

which can be measured using the static and dynamic techniques. This experimental scheme promises a record sensitivity to force differences at the level of 1 fN. In the second experiment, we propose to demonstrate the modulation of the Casimir force by optically switching the insulator-metal transition in VO₂ films [26]. The phase transition between the insulator and metal leads to a change in the charge carrier density of order 10^4 , which is sufficient to bring about a large change in the Casimir force. For both experiments the related theory is elaborated and the magnitudes of Casimir forces are computed with application to the experimental configurations. The effects from using different theoretical approaches to the description of conductivity processes are carefully analyzed and shown to be observable in the proposed experiments. The present status of developing the apparatus at UC Riverside, its calibration and sensitivity is presented. The proposed experiments offer a precision test of the role of conductivity, optical properties and scattering in the Lifshitz theory of the van der Waals and Casimir force at nonzero temperature. They also open up possibilities of radically new nanomechanical devices using the Casimir force in imaging applications.

The paper is organized as follows. In Sec. II, we describe the proposed experiment on the difference Casimir force with the patterned semiconductor plate. The brief description of the experimental apparatus and preliminary results are also provided. Section III contains the calculation of the difference Casimir force and equivalent pressure in the patterned geometry using the Lifshitz theory at nonzero temperature and different models of conductivity processes at low frequencies. In Sec. IV we propose the experiment on the modulation of the Casimir force through a metal-insulator transition. The experimental scheme and some preliminary tests are discussed. Section V presents theoretical computations of Casimir forces in a insulator-metal transition on the basis of the Lifshitz theory at nonzero temperature and using different models for the conductivity processes. Section VI contains our conclusions and a discussion.

II. PROPOSED EXPERIMENT ON THE DIFFERENCE CASIMIR FORCE WITH THE PATTERNED SEMICONDUCTOR PLATE

The aim of this experiment is to gain a fundamental understanding of the role of carrier density in the Casimir force using a nanofabricated patterned semiconductor plate. The proposed design for the experiment is shown schematically in Fig. 1. The gold coated

polystyrene sphere of about $100\text{ }\mu\text{m}$ radius is attached to a cantilever of an atomic force microscope (AFM) specially adapted for making sensitive force measurements. Instead of the simple single crystal Si substrate used in the previous experiments [23, 24], here a patterned Si plate is employed. This plate is composed of single crystal Si specifically fabricated to have adjacent sections of two different charge carrier densities $n \sim 10^{16}\text{ cm}^{-3}$ and $\tilde{n} \sim 10^{20}\text{ cm}^{-3}$. In this range of doping densities, the plasma frequency ω_p will change by a factor of 100. Additional changes in the ω_p can be brought about by using both $p(\text{B})$ and $n(\text{P})$ type dopants as electrons and holes differ in their effective mass by 30%.

The preparation of the Si sample with the two sections having different conductivities is shown schematically in Fig. 2. First, one half of the bare Si wafer of 0.3 to 0.5 mm thickness [23, 24] having the lower conductivity shown in (a) is masked with a photoresist as shown in (b). Next in (c) the exposed half of the Si wafer is doped with P ions using ion implantation leading to a higher density of electrons in the exposed half in (d). Rapid thermal annealing and chemical mechanical polishing of the patterned Si plate will be done as the last step. This is to ensure that there are no surface features resulting from the fabrication. Similar nanofabrication procedures of semiconductors were used in our previous work [24, 27, 28]. Sharp transition boundaries between the two sections of Si plate of width less than 200 nm are possible. The limitation comes from interdiffusion and the resolution of the ion implantation procedure to be used. It might be necessary to further limit interdiffusion by the creation of a narrow 100 nm barrier between the two doped regions.

Identically prepared but unpatterned samples will be used to measure the properties which are needed for theoretical computations. The carrier concentration will be measured using Hall probes. This will yield an independent measurement of the plasma frequencies. A four probe technique will be used to measure the conductivity σ . From the conductivity and the charge carrier concentration, the scattering time $\tau = \sigma/(\varepsilon_0\omega_p^2)$ will be found, where ε_0 is the permittivity of free space.

The first important improvement of this experiment, as compared with Refs. [23, 24], is the direct measurement of difference forces when the patterned plate is oscillated below the sphere. This measurement is performed as follows. The patterned Si plate will be mounted on the piezo below a Au coated sphere as is shown in Fig. 1. The Si plate is positioned such that the boundary is below the vertical diameter of the sphere. The distance between the sphere and Si plate z will be kept fixed and the Si plate will be oscillated in

the horizontal direction using the piezo such that the sphere crosses the boundary in the perpendicular direction during each oscillation (a similar approach was exploited in Ref. [29] for constraining new forces from the oscillations of the Au coated sphere over two dissimilar metals, Au and Ge). The Casimir force on the sphere changes as the sphere crosses the boundary. This change corresponds to the differential force

$$\Delta F(z) = F_{\tilde{n}}(z) - F_n(z), \quad (1)$$

equal to the difference of the Casimir forces due to two different charge carriers densities \tilde{n} and n , respectively. This causes a difference in the deflection of the cantilever. In order to reduce the random noise by averaging, the periodic horizontal movement of the plate will be of an angular frequency $\Omega \sim 0.1$ Hz. The amplitude of the plate oscillations is limited by the piezo characteristics, but will be of order $100 \mu\text{m}$, much larger than typical transition region of 200 nm . The experiment will be repeated for different sphere-plate separations in the region from 100 to 300 nm . The measurement of absolute separations will be performed by the application of voltages to the test bodies as described in Ref. [23].

The second major improvement in this experiment in comparison with all previous measurements of the Casimir force is the increased sensitivity. This will be achieved through the use of the interferometer based low temperature AFM capable of operating over wide temperature range spanning from 360 to 4 K , and the use of two measurement techniques, a static one and a dynamic one. A picture of the newly constructed experimental apparatus of a low-temperature AFM is shown in Fig. 3. Here the cantilever deflection is measured interferometrically and therefore has much higher sensitivity than photodiodes used in the previous work [23, 24, 25]. The detection of a difference force $\Delta F(z)$ will be done by two alternative techniques. The first technique, a static one, reduces to the direct measurement of $\Delta F(z)$ as described above. We are presently performing the initial tests and calibration trials at 77 K . An oil free vacuum with a pressure of around $2 \times 10^{-7} \text{ Torr}$ is used. The instrument is magnetically damped to yield low mechanical coupling to the environment. The temperature can be varied with a precision of 0.2 K . We have fabricated special conductive cantilevers with a spring constant $k = 0.03 \text{ N/m}$. The magnitude of k is found by applying electrostatic voltages to the plate as discussed in Ref. [23]. To accomplish this, Si cantilevers were thermal diffusion doped to achieve the necessary conductivity. Note that conductive cantilevers are necessary to reduce electrostatic effects. The cantilever-sphere arrangement

has been checked to be stable at 77 K.

The experimental setup using the static measurement technique allows not only a demonstration but a detailed investigation of the influence of carrier density, conductivity and scattering in semiconductors on the Casimir force. According to the results of Refs. [23, 24, 25] and calculations below in Sec. III, the magnitude of the difference force to be measured is about several pN. Different theoretical models of conductivity processes at low frequencies lead to predictions differing by approximately 1 pN within a wide separation region (see Fig. 5 in Sec. III). The described setup provides excellent opportunity for precise measurement of forces of order and below 1 pN. We have measured a resonance frequency of the cantilever of $f_r = 1130.9$ Hz, a quality factor $Q = 5889.2$, and an equivalent noise bandwidth $B = 0.3$ Hz. The resultant force sensitivity of our cantilever at $T = 77$ K with the gold coated sphere attached was determined following Ref. [30] to be

$$\delta F_{\min} = \left(\frac{2k_B T k B}{\pi Q f_r} \right)^{1/2} \approx 0.96 \times 10^{-15} \text{ N} \approx 1 \text{ fN}, \quad (2)$$

where k_B is the Boltzmann constant. Even bearing in mind that the systematic error may be up to an order of magnitude larger, the sensitivity (2) presents considerable possibilities for the precise investigation of the difference Casimir force ΔF .

The second technique for the detection of the difference Casimir force is a dynamic one [7, 8, 21]. Here this technique is applied not for a direct measurement of ΔF but rather for the experimental determination of the equivalent difference Casimir pressure between the two parallel plates (one made of Au and the other one, a patterned Si plate). The cantilever-sphere system oscillates in the vertical direction due its thermal noise with a resonant frequency $\omega_r = (k/M)^{1/2} = 2\pi f_r$ in the absence of the Casimir force, where M is the mass of the system. The thermal noise spectrum of the sphere-cantilever system is measured and fit to a Lorentzian to identify the peak resonant frequency, ω_r . The shift of ω_r in the presence of the Casimir force when for example the sphere is positioned above a section of the patterned Si plate with the density of charge carriers \tilde{n} is equal to [7, 8, 21]

$$\omega_{r,\tilde{n}} - \omega_r = -\frac{\omega_r}{2k} \frac{\partial F_{\tilde{n}}(z)}{\partial z}. \quad (3)$$

Next the plate is oscillated in the horizontal direction with a frequency Ω . As a result, the frequency shift

$$\omega_{r,\tilde{n}} - \omega_{r,n} = -\frac{\omega_r}{2k} \frac{\partial \Delta F(z)}{\partial z} \quad (4)$$

between the resonant frequencies above the two different sections of the patterned Si plate is measured. Using the proximity force approximation [4, 31], we determine the difference Casimir pressure

$$\Delta P(z) = -\frac{1}{2\pi R} \frac{\partial \Delta F(z)}{\partial z} \quad (5)$$

between the two parallel plates (the Au one and the patterned Si). Note that the systematic error from the use of the proximity force approximation was recently confirmed to be less than z/R [32, 33, 34, 35, 36]. Equations (4) and (5) express the difference Casimir pressure through the measured shift of the resonance frequency above the two halves of the patterned plate. As is shown in the next section, the measurements of the difference Casimir pressure using the dynamic technique provides us with one more test of the predictions of the different models of conductivity processes at low frequencies.

The experiment on the difference Casimir force from a patterned Si plate, as described in this section, allows variation of charge carrier density by the preparation of different semiconductor samples. Thus, the proposed measurements should provide a comprehensive understanding on the role of conductivity and optical processes in the Casimir force for nonmetallic materials and discriminate between competing theoretical approaches.

III. CALCULATION OF THE DIFFERENCE CASIMIR FORCE AND PRESSURE IN THE PATTERNED GEOMETRY

The difference Casimir force and the equivalent Casimir pressure from the oscillation of the patterned Si plate below an Au coated sphere at $T = 300$ K in thermal equilibrium are given by the Lifshitz theory. In the static technique the data to be compared with theory is the difference of Casimir forces acting between the sphere and two sections of the patterned plate. This difference is obtained from the Casimir energy between two parallel plates, as given by the Lifshitz theory, using the proximity force approximation [2, 3, 4]

$$\begin{aligned} \Delta F(z) = k_B T R \sum_{l=0}^{\infty} \left(1 - \frac{1}{2} \delta_{l0}\right) \int_0^{\infty} k_{\perp} dk_{\perp} \\ \times \ln \frac{[1 - r_{TM;\tilde{n}}(\xi_l, k_{\perp}) r_{TM}(\xi_l, k_{\perp}) e^{-2q_l z}][1 - r_{TE;\tilde{n}}(\xi_l, k_{\perp}) r_{TE}(\xi_l, k_{\perp}) e^{-2q_l z}]}{[1 - r_{TM;n}(\xi_l, k_{\perp}) r_{TM}(\xi_l, k_{\perp}) e^{-2q_l z}][1 - r_{TE;n}(\xi_l, k_{\perp}) r_{TE}(\xi_l, k_{\perp}) e^{-2q_l z}]} \end{aligned} \quad (6)$$

Here $\xi_l = 2\pi k_B T l / \hbar$ with $l = 0, 1, 2, \dots$ are the Matsubara frequencies, $q_l = (k_{\perp}^2 + \xi_l^2 / c^2)^{1/2}$, and k_{\perp} is the projection of the wave vector on the boundary planes. The reflection coefficients

on the Au plane for the two independent polarizations of the electromagnetic field (transverse magnetic and transverse electric modes) are

$$r_{TM}(\xi_l, k_\perp) = \frac{\varepsilon_l q_l - k_l}{\varepsilon_l q_l + k_l}, \quad r_{TE}(\xi_l, k_\perp) = \frac{k_l - q_l}{k_l + q_l}, \quad (7)$$

where $k_l = (k_\perp^2 + \varepsilon_l \xi_l^2 / c^2)^{1/2}$ and $\varepsilon_l = \varepsilon(i\xi_l)$ is the dielectric permittivity of Au along the imaginary frequency axis. In a similar way, the reflection coefficients on the two sections of a patterned Si plate with charge carrier densities \tilde{n} and n are given, respectively, by

$$r_{TM;\tilde{n},n}(\xi_l, k_\perp) = \frac{\varepsilon_{l;\tilde{n},n} q_l - k_{l;\tilde{n},n}}{\varepsilon_{l;\tilde{n},n} q_l + k_{l;\tilde{n},n}}, \quad r_{TE;\tilde{n},n}(\xi_l, k_\perp) = \frac{k_{l;\tilde{n},n} - q_l}{k_{l;\tilde{n},n} + q_l}, \quad (8)$$

where $k_{l;\tilde{n},n} = (k_\perp^2 + \varepsilon_{l;\tilde{n},n} \xi_l^2 / c^2)^{1/2}$ and $\varepsilon_{l;\tilde{n},n} = \varepsilon_{\tilde{n},n}(i\xi_l)$ are the dielectric permittivities of Si with charge carrier densities \tilde{n} and n along the imaginary frequency axis.

In the dynamic technique the data to be compared with theory is the equivalent difference Casimir pressure between two parallel plates, one made of Au and the other one a patterned Si plate. Using the same notations as above, the difference Casimir pressure is given by

$$\begin{aligned} \Delta P(z) = & -\frac{k_B T}{\pi} \sum_{l=0}^{\infty} \left(1 - \frac{1}{2} \delta_{l0}\right) \int_0^\infty k_\perp dk_\perp q_l \\ & \times \left\{ [r_{TM;\tilde{n}}^{-1}(\xi_l, k_\perp) r_{TM}^{-1}(\xi_l, k_\perp) e^{2q_l z} - 1]^{-1} + [r_{TE;\tilde{n}}^{-1}(\xi_l, k_\perp) r_{TE}^{-1}(\xi_l, k_\perp) e^{2q_l z} - 1]^{-1} \right. \\ & \left. - [r_{TM;n}^{-1}(\xi_l, k_\perp) r_{TM}^{-1}(\xi_l, k_\perp) e^{2q_l z} - 1]^{-1} - [r_{TE;n}^{-1}(\xi_l, k_\perp) r_{TE}^{-1}(\xi_l, k_\perp) e^{2q_l z} - 1]^{-1} \right\}. \end{aligned} \quad (9)$$

Note that in Eqs. (6) and (9) we have replaced the 100 nm Au coating and the 0.3–0.5 mm Si plate for an Au and Si semispaces, respectively. Using the Lifshitz formula for layered structures [4] it is easy to calculate the force and pressure errors due to this replacement. For example, for an Au layer at a typical separation of 100 nm this error is about 0.01%. For Si a finite thickness of the plate d markedly affects the Casimir force when the separation distance z exceeds the thickness, i.e., $z/d > 1$ [37]. In our case, however, even at the largest separation considered ($z = 300$ nm) the ratio of the separation to the plate thickness $z/d \leq 10^{-3}$. This is similar to the case of the experiment [25] where the finite thickness of Si membrane also does not influence the magnitude of the Casimir force because at separations $z \leq 200$ nm where statistically meaningful results were obtained $z/d \leq 0.05$.

We have performed computations of the difference Casimir force (6) and difference Casimir pressure (9) for samples with typical values of charge carrier concentrations \tilde{n} and n as used in experiments [23, 24, 25]. Both sections of the Si plate were chosen to have

electron conductivity and doped with P. For the section of the plate with higher concentration of charge carriers the values $\tilde{n}_1 = 3.2 \times 10^{20} \text{ cm}^{-3}$ (such a sample was fabricated in Ref. [24]) and $\tilde{n}_2 = 3.2 \times 10^{19} \text{ cm}^{-3}$ were used in the computations. The respective dielectric permittivity along the imaginary frequency axis can be represented in the form [38]

$$\varepsilon_{\tilde{n}}(i\xi_l) = \varepsilon^{Si}(i\xi_l) + \frac{\omega_{p;\tilde{n}}^2}{\xi_l(\xi_l + \gamma_{\tilde{n}})}. \quad (10)$$

Here $\varepsilon^{Si}(i\xi_l)$ is the permittivity of high-resistivity (dielectric) Si along the imaginary frequency axis computed in Ref. [39] by means of the dispersion relation using the tabulated optical data for the complex index of refraction [40]. The values of the plasma frequencies and relaxation parameters are the following [24]: $\omega_{p;\tilde{n}_1} = 2.0 \times 10^{15} \text{ rad/s}$, $\gamma_{\tilde{n}_1} = 2.4 \times 10^{14} \text{ rad/s}$, $\omega_{p;\tilde{n}_2} = 6.3 \times 10^{14} \text{ rad/s}$, $\gamma_{\tilde{n}_2} = 1.8 \times 10^{13} \text{ rad/s}$. In Fig. 4 the dielectric permittivities of the samples with high concentrations of charge carriers \tilde{n}_1 and \tilde{n}_2 are shown as solid lines 1 and 2, respectively. In the same figure, the dashed line *a* shows the permittivity of high-resistivity, dielectric, Si [39] and the dotted line the permittivity of Au computed in Ref. [39] using the tabulated optical data of Ref. [40].

Below we will use two models for the permittivity of the section of the Si plate with lower concentration of charge carriers n . Calculations show that for any $0 < n \leq 1.0 \times 10^{17} \text{ cm}^{-3}$ (this interval includes the experimental value of $n \approx 1.2 \times 10^{16} \text{ cm}^{-3}$ in Ref. [24]), the obtained values of $F_n(z)$ and, thus, of $\Delta F(z)$ do not depend on n . Because of this we use in the computations $n = 1.0 \times 10^{17} \text{ cm}^{-3}$, the plasma frequency $\omega_{p;n} = 3.5 \times 10^{13} \text{ rad/s}$ and the relaxation parameter $\gamma_n = 1.8 \times 10^{13} \text{ rad/s}$ [24, 41] (note that for $n \leq 1.0 \times 10^{17} \text{ cm}^{-3}$ the value of the relaxation parameter does not effect the magnitude of the Casimir force). Then the dielectric permittivity of this section of the Si plate along the imaginary frequency axis is given by

$$\varepsilon_n^{(b)}(i\xi_l) = \varepsilon^{Si}(i\xi_l) + \frac{\omega_{p;n}^2}{\xi_l(\xi_l + \gamma_n)} \quad (11)$$

and is shown as the dashed line *b* in Fig. 4. This is one model of Si with a lower concentration of charge carriers referred to below as model (*b*).

As is seen in Fig. 4, the dashed line *b*, and thereby all respective lines for samples with the concentration of charge carriers smaller than $1.0 \times 10^{17} \text{ cm}^{-3}$, deviate from the permittivity of dielectric Si (line *a*) only at frequencies below the first Matsubara frequency ξ_1 . Because of this, it is common (see, e.g., [2, 3, 42]) to neglect the small conductivity of high-resistivity materials at low frequencies and describe them in the frequency region below the

first Matsubara frequency by the static dielectric permittivity. In our case this leads to

$$\varepsilon_n^{(a)}(i\xi_l) = \varepsilon^{Si}(i\xi_l), \quad (12)$$

which is the other model for Si with a lower concentration of charge carriers referred to below as model (a). From Eq. (12) and Fig. 4 at all frequencies $\xi \leq \xi_1$ it follows: $\varepsilon_n^{(a)}(i\xi) = \varepsilon^{Si}(0) = 11.66$.

To be exact, at any $T > 0$ the density of free charge carriers n in semiconductors (and even in dielectrics) and thus the conductivity are nonzero ($n > 0$). Thus, the model (11) should be considered as more exact than the model (12). At the same time, if we note that for $n \leq 1.0 \times 10^{17} \text{ cm}^{-3}$ the conductivity is small, it should be expected that both models should lead to practically identical results. This is, however, not so. In Fig. 5 we present the computational results for the difference Casimir force using Eq. (6). The solid line 1a demonstrates the values of the difference Casimir force versus separation for the patterned Si plate with a higher concentration of charge carriers \tilde{n}_1 computed under the assumption that the lower concentration section of the plate is described by Eq. (12), i.e., the conductivity processes at low frequencies are neglected. The dashed line 1b shows the difference Casimir force as a function of separation computed with the same \tilde{n}_1 but taking into account the conductivity processes at low frequencies in accordance with Eq. (11). As is seen from the comparison of lines 1a and 1b, the difference Casimir forces computed using Eqs. (12) and (11) differ by 1.2 pN at a separation $z = 100 \text{ nm}$ and this difference slowly decreases to approximately 0.14 pN at a separation $z = 300 \text{ nm}$. The lines 2a and 2b present similar results for the case when the higher charge carrier density is equal to \tilde{n}_2 . As is seen from Fig. 5, decreasing the higher concentration by an order of magnitude decreases the predicted magnitude of the difference Casimir force by more than two times, but leaves the same gap between the predictions of two different models of the permittivity at low frequencies.

Importantly, our predictions do not depend on the discussions mentioned in the Introduction on the optical properties of metals at quasi-static frequencies [10, 11, 12, 13, 14, 15, 16, 17]. The resolution of this controversy affects only the value of the Au reflection coefficient $r_{TE}(0, k_\perp)$ at zero frequency. The latter, however, does not contribute to the difference Casimir force (6) and pressure (9) because for dielectrics and semiconductors $r_{TE;\tilde{n},n}(0, k_\perp) = 0$ regardless of what model (11) or (12) is used for the description of the dielectric permittivity at low frequencies. The obtained difference between the lines 1a – 1b

and $2a - 2b$ in Fig. 5 is completely explained by the different contributions of semiconductor reflection coefficient $r_{TM;n}(0, k_\perp)$ when one uses Eq. (11) or Eq. (12) to describe the dielectric permittivity at low frequencies. Regarding the semiconductor section with a higher charge carrier density \tilde{n} , from Eqs. (8), (10) it is always valid that

$$r_{TM;\tilde{n}}(0, k_\perp) = 1. \quad (13)$$

However, for the section of the plate with a lower charge carrier density n it follows from Eq. (8) that

$$r_{TM;n}(0, k_\perp) = 1 \quad \text{or} \quad r_{TM;n}(0, k_\perp) = \frac{\varepsilon^{Si}(0) - 1}{\varepsilon^{Si}(0) + 1} \quad (14)$$

when Eq. (11) or Eq. (12) are used, respectively. Thus, the difference between the lines $1a$ and $1b$ (and the same difference between the lines $2a$ and $2b$) can be found analytically. Taking only the zero-frequency contribution in Eq. (6) and subtracting the difference Casimir force calculated using Eq. (11) [model (b)] from the difference Casimir force calculated using Eq. (12) [model (a)] one obtains

$$\Delta F_a^{(0)} - \Delta F_b^{(0)} = -\frac{k_B T R}{8z^2} \left\{ \zeta(3) - \text{Li}_3 \left[\frac{\varepsilon^{Si}(0) - 1}{\varepsilon^{Si}(0) + 1} \right] \right\}. \quad (15)$$

Here $\zeta(z)$ is the Riemann zeta function, and $\text{Li}_3(z)$ is the polylogarithm function. The results using the analytic Eq. (15) coincide with the differences between the lines $1a - 1b$ and $2a - 2b$ in Fig. 5 computed numerically.

In the experiment [25] the difference Casimir force between Au coated sphere and Si plate illuminated with laser pulses was first measured. In the presence of light the charge carrier density was about $2 \times 10^{19} \text{ cm}^{-3}$ and in the absence of light of about $5 \times 10^{14} \text{ cm}^{-3}$. The experimental data were shown to be in agreement with model (a) which uses the finite static dielectric permittivity of Si. The model (b) which includes the dc conductivity of Si was excluded at 95% confidence within the separation region from 100 to 200 nm. As was discussed above, in the framework of the Lifshitz theory this result is rather unexpected. Bearing in mind that illumination with laser pulses leads to several additional sources of errors discussed in Ref. [25], it is of vital interest to verify the obtained conclusions in a more precise experiment with patterned Si plates. The comparison of the experimental sensitivities presented in Sec. II with the magnitudes of the difference Casimir forces computed here using different theoretical models demonstrate that the proposed experiment with patterned

semiconductor plate will bring decisive results on the discussed problems in the Lifshitz theory at nonzero temperature.

The calculations of the difference Casimir pressure determined in the dynamic mode of the proposed experiment leads to results analogous to those for the difference force. The calculation results using Eq. (9) with the same values of parameters as above and two models of lower conductivity Si are presented in Fig. 6. Here the difference Casimir pressures between an Au plate and a patterned Si plate with the higher densities of charge carriers $\tilde{n}_{1,2}$ (one section of the plate) and lower n (another section of the plate) are shown with solid lines 1*a* and 2*a*, respectively, computed under the assumption that Si with the lower n possesses a finite permittivity (12) at zero frequency. The dashed lines 1*b* and 2*b* are obtained under the assumption that Si with the lower n is described by the permittivity (11) which goes to infinity when the frequency goes to zero. As is seen in Fig. 6, the difference Casimir pressure with a patterned plate with charge carrier densities \tilde{n}_1 and n equals 250 mPa at a separation $z = 100$ nm [model (a) of low conductivity section of the plate] and the difference in predictions for the two models equals 38.6 mPa. The proposed experiment of the difference Casimir pressure can reliably discriminate between the solid and dashed lines in Fig. 6 thus providing one more test for the Lifshitz theory at nonzero temperature.

Notice that in a similar way to the force, the differences between the lines 1*a* – 1*b* and 2*a* – 2*b* in Fig. 6 are expressed analytically by taking the zero-frequency contributions in Eq. (9):

$$\Delta P_a^{(0)} - \Delta P_b^{(0)} = -\frac{k_B T}{8\pi z^3} \left\{ \zeta(3) - \text{Li}_3 \left[\frac{\varepsilon^{Si}(0) - 1}{\varepsilon^{Si}(0) + 1} \right] \right\}. \quad (16)$$

Calculations using Eq. (16) lead to the same differences between the lines 1*a* – 1*b* and 2*a* – 2*b* as were computed numerically in Fig. 6.

IV. PROPOSED EXPERIMENT ON THE MODULATION OF THE CASIMIR FORCE THROUGH AN INSULATOR-METAL TRANSITION

The exciting possibility for the modulation of the Casimir force due to a change of charge carrier density is offered by semiconductor materials that undergo the insulator-metal transition with the increase of temperature. Such a transition leads to a change of the carrier density of order 10^4 . Although in literature it is common to speak about insulator-metal transition, this can be considered as a transformation between two semiconductor phases

with lower and higher charge carrier densities n and \tilde{n} , respectively. As was shown above, this is sufficient to bring about a large change in the Casimir force. From a fundamental point of view, the modulation of the Casimir force due to the phase transition will offer one more precision test of the role of conductivity and optical properties in the Lifshitz theory of the Casimir force. This experiment suggests some advantages as compared to the difference force measurement with a patterned plate considered in Secs. II and III. First, because of the large change in the magnitude and bandwidth of the optical properties in a phase transition, the modulation of the Casimir force will be larger. Second, an insulator-metal transition does not require the special fabrication of patterned plates with one section having a high carrier density, which might not be compatible with robust device design. Keeping in mind that the increase of temperature necessary for the phase transition can be induced by laser light, this opens up the possibility of radically new nanomechanical devices using the Casimir force in image detection. The phase transition can be also brought about through electrical heating of the material.

In this experiment we propose to measure the change of the Casimir force acting between an Au coated sphere and a vanadium dioxide (VO_2) film deposited on sapphire substrate which undergoes the insulator-metal transition with the increase of temperature. It has been known that VO_2 crystals and thin films undergo an abrupt transition from semiconducting monoclinic phase at room temperature to a metallic tetragonal phase at 68°C [26, 43, 44, 45, 46, 47, 48]. The phase transition causes the resistivity of the sample to decrease by a factor of 10^4 from $10\,\Omega\text{cm}$ to $10^{-3}\,\Omega\text{cm}$ (i.e., the same change as for two semiconductor half plates in Sec. II with lower and higher charge carrier densities). In addition, the optical transmission for a wide region of wavelengths extending from $1\,\mu\text{m}$ to greater than $10\,\mu\text{m}$, decreases by more than a factor of 10–100.

The schematic of the experimental setup is shown in Fig. 7. In this figure, light from a chopped 980 nm laser will be used to heat the VO_2 film [43, 44]. About 10–100 mW power of the 980 nm laser is required to bring about all optical switching of VO_2 films. The same procedure as outlined in Sec. II (the static technique) will be used in the measurement of the modulation of the Casimir force including the interferometric detection of cantilever deflection. The schematic of the setup is similar to the one used in Ref. [25] in the demonstration of optically modulated dispersion forces. An important point is that in Ref. [25] the absorption of light from a 514 nm Ar laser led to an increase of charge carrier density.

By contrast, here the wavelength of a laser is selected in such a way that light only leads to heating of a VO_2 film but does not change the number of free charge carriers [43].

As a first step towards studying the role of the insulator-metal transition in the Casimir force, we have recently fabricated thin films of VO_2 on sapphire plates. The preliminary results are shown in Fig. 8. It is observed that we have obtained more than a factor of 10 change in the resistivity of the film. These films were prepared by thermal evaporation of VO_2 powder. While films of appropriate thickness approaching 100 nm and roughness of about 2 nm (shown in Fig. 9) can be obtained by this procedure, it is not optimal as it leads to the non-stoichiometric formation of the mixed valence states of the vanadium oxide (VO_x). In the future rf magnetron sputtering will be used to make the films [43]. VO_2 films using this technique have been shown to have the 10^4 change in resistivity and a corresponding large change in optical reflectivity and spectrum.

The aim of the proposed experiment on the influence of insulator-metal transition on the Casimir force is two fold: applications for actuation of nanodevices through a modulation of the Casimir force, and to perform fundamental tests on the theory of dispersion forces. To accomplish this, two types of measurements are planned. In the first we plan to demonstrate the modulation of the Casimir force through an optical switching of the insulator-metal transition. This modulation will lead to novel microdevices as optical and electrical switches, optical modulators, optical filters and IR detectors that can be actuated optically through the absorption of IR radiation. Importantly, such devices can be integrated with Si technology which is used in the fabrication of microelectromechanical systems [43]. In the second type of measurements, the variable temperature atomic force microscope described in Sec. II will be used to perform precision measurements of the Casimir force between a gold coated sphere and VO_2 film. Here, the Casimir force will be measured at different temperatures from room temperature through 80°C . This temperature range spans the dielectric (semiconducting) and metal regions of VO_2 . Careful comparison of the experimental data and the theory (see the next section) will be done to understand the role of conductivity and losses in both phases of VO_2 .

V. CALCULATION OF THE CASIMIR FORCE IN AN INSULATOR-METAL TRANSITION

The Casimir force acting between a metal coated sphere and the VO₂ film on a sapphire plate both before and after the phase transition (i.e., in the insulating and metal phases or, more exactly, in the semiconductor phases with lower and higher charge carrier densities) are expressed by the Lifshitz formulas in accordance with Eqs. (1) and (6). As above, we label the higher concentration of charge carriers \tilde{n} and the lower concentration n . To compute the Casimir force before and after the phase transition one needs the optical properties of VO₂ on a sapphire plate in a wide frequency region.

In Ref. [47] the dielectric permittivity of VO₂ is measured and fitted to the oscillator model for both bulk VO₂ and for a system of 100 nm thick VO₂ film deposited on bulk sapphire plate within the frequency region from 0.25 eV to 5 eV. This modelling was performed both before and after the phase transition. Typical thickness of sapphire substrate is of about 0.3 mm, i.e., the same as the thickness of patterned Si plate in Secs. II and III. Because of this, when calculating the Casimir force between gold coated sphere and VO₂ film on sapphire substrate, we can use the Lifshitz formula for bulk test bodies (see Sec. III for details). The application region of the models presented in Ref. [47] should be extended in order to perform computations of the Casimir force within the separation region from 100 to 300 nm where contribution from optical data up to about 10 eV have to be taken into account. For this purpose, we have supplemented equations of Ref. [47] with additional terms taking into account the frequency-dependent electronic transitions at high frequencies [49, 50]. As a result, the effective dielectric permittivity of the VO₂ film on a sapphire substrate before the phase transition (at $T = 300$ K) is given by

$$\varepsilon_n(i\xi_l) = 1 + \sum_{i=1}^7 \frac{s_{n,i}}{1 + \frac{\xi_l^2}{\omega_{n,i}^2} + \Gamma_{n,i} \frac{\xi_l}{\omega_{n,i}}} + \frac{\varepsilon_\infty^{(n)} - 1}{1 + \frac{\xi_l^2}{\omega_\infty^2}}. \quad (17)$$

Here the values of the oscillator frequencies $\omega_{n,i}$, dimensionless relaxation parameters $\Gamma_{n,i}$ and of the oscillator strengths $s_{n,i}$ taken from Fig. 5 in Ref. [47] are presented in Table I. The constants related to the contribution of high-frequency electronic transitions [the last term on the right-hand side of Eq. (17)] are $\varepsilon_\infty^{(n)} = 4.26$ [47] and $\omega_\infty = 15$ eV. If we put $\xi_l = 0$ in the last term on the right-hand side of Eq. (17), this equation is the same as the result in Ref. [47].

After the phase transition we have a phase with increased charge carrier density \tilde{n} . Similar to Eq. (10) the effective dielectric permittivity of the VO₂ film on a sapphire substrate can be described by the dielectric permittivity

$$\begin{aligned} \varepsilon_{\tilde{n}}(i\xi_l) = & 1 + \frac{\omega_{p;\tilde{n}}^2}{\xi_l(\xi_l + \gamma_{\tilde{n}})} \\ & + \sum_{i=1}^4 \frac{s_{\tilde{n},i}}{1 + \frac{\xi_l^2}{\omega_{\tilde{n},i}^2} + \Gamma_{\tilde{n},i} \frac{\xi_l}{\omega_{\tilde{n},i}}} + \frac{\varepsilon_{\infty}^{(\tilde{n})} - 1}{1 + \frac{\xi_l^2}{\omega_{\infty}^2}}. \end{aligned} \quad (18)$$

Parameters $\omega_{\tilde{n},i}$, $\Gamma_{\tilde{n},i}$ and $s_{\tilde{n},i}$ can be found in Fig. 6 of Ref. [47] and are listed in Table II. The other parameters are $\varepsilon_{\infty}^{(\tilde{n})} = 3.95$, $\omega_{p;\tilde{n}} = 3.33$ eV, $\gamma_{\tilde{n}} = 0.66$ eV [47]. Setting $\xi_l = 0$ in the last term on the right-hand side of Eq. (18) (this term takes high-frequency electronic transitions into account), returns (18) to the original form suggested in Ref. [47]. Note that the recently suggested model for the dielectric permittivity of VO₂ films [51] is applicable not only before and after a phase transition but also at intermediate temperatures. This model is, however, restricted to a more narrow frequency region from 0.73 to 3.1 eV and uses the simplified description of two oscillators before the phase transition and only one oscillator with nonzero frequency after it.

In Fig. 10, the effective dielectric permittivity of VO₂ film of 100 nm thickness on sapphire substrate before and after the phase transition, as given in Eqs. (17) and (18) is shown by the solid lines 1 and 2, respectively. In the same figure, the dielectric permittivity of Au versus frequency is shown as dots. The vertical line indicates the position of the first Matsubara frequency at $T = 340$ K (i.e., in the region of the phase transition).

In Fig. 11 we present the computational results for the Casimir force between the Au coated sphere and VO₂ film on sapphire substrate versus separation obtained by the substitution of the dielectric permittivity (17) (VO₂ before the phase transition in solid line 1) and (18) (VO₂ after the phase transition in solid line 2) into the Lifshitz formula. As is seen in Fig. 11, after the phase transition the magnitudes of the Casimir force increase due to an increase in the charge carrier density. For a comparison with the proposed experiment on the difference Casimir force from a patterned Si plate, in Fig. 12 (solid line) we plot the difference of the Casimir forces after and before the phase transition, i.e., the difference of lines 2 and 1 in Fig. 11. It is seen that the difference Casimir force from a phase transition changes from 13 pN at $z = 100$ nm to 1.2 pN at $z = 300$ nm, i.e., the magnitudes of the difference from the phase transition are greater than that from the patterned Si plate.

The difference Casimir force in the insulator-metal phase transition provides us with one more test on the proper modelling of the dielectric permittivity in the Lifshitz theory of dispersion forces. Similar to Sec. III, we arrive at different results for the difference Casimir force after and before the phase transition if the conductivity of a dielectric VO₂ at zero frequency is taken into account in our computations. The shift in the values of the difference Casimir force is completely determined by the change of the zero-frequency term in the Lifshitz formula. By analogy with Eq. (15) it follows:

$$\Delta F_a^{(0)} - \Delta F_b^{(0)} = -\frac{k_B T R}{8z^2} \left\{ \zeta(3) - \text{Li}_3 \left[\frac{\varepsilon^{\text{VO}_2}(0) - 1}{\varepsilon^{\text{VO}_2}(0) + 1} \right] \right\}, \quad (19)$$

where b represents the case when the dc conductivity of an insulating VO₂ is taken into account, and a represents the case when insulating VO₂ is described by the permittivity (17). From Eq. (17) and Table I one obtains

$$\varepsilon^{\text{VO}_2}(0) \equiv \varepsilon_n(0) = \varepsilon_\infty^{(n)} + \sum_{i=1}^7 s_{n,i} = 9.909. \quad (20)$$

In Fig. 12 the difference Casimir force between an Au coated sphere and VO₂ film on sapphire substrate after and before the phase transition computed including the dc conductivity of insulating VO₂ versus separation is plotted with the dashed line. The difference between the solid and dashed lines is determined by Eq. (19). This difference changes from 1.6 pN at $z = 100$ nm to 0.2 pN at $z = 300$ nm. Thus, in the phase transition experiment the predicted discrepancies between the two theoretical approaches to the description of conductivity properties at low frequencies are larger than in the experiment with the patterned semiconductor plate. This will help to experimentally discriminate between the two approaches and deeply probe the role of the material properties in the Lifshitz theory at nonzero temperature.

VI. CONCLUSIONS AND DISCUSSION

In the above we have proposed two experiments on the measurement of the difference Casimir force acting between a metal coated sphere and a semiconductor with different charge carrier densities. One of these experiments is based on the formation of a special patterned Si plate, two sections of which have charge carrier densities differing by several orders of magnitude. The measurement scheme in this experiment is differential, i.e., adapted for the direct measurement of the difference in the Casimir forces between the sphere and

each section of the patterned plate. This allows one to obtain high precision within a wide measurement range. Using the dynamic measurement technique, this experiment also permits the measurement of the difference Casimir pressure between two parallel plates one of which is coated with gold and the other is patterned and consists of two sections with different charge carrier densities.

Another proposed experiment directed to the same objective is novel and uses the insulator-metal phase transition brought about by an increase of temperature in the measurements of the Casimir force. This transition also leads to the change of charge carrier density by several orders of magnitude while not requiring the formation of special patterned samples. The expected difference in the Casimir forces after and before the phase transition is even larger than in the experiment with the patterned Si plate.

Both proposed experiments are motivated by the uncertainties in the application of the theory of dispersion forces at nonzero temperature. As was shown above, different models of the conductivity of semiconductors at low frequencies used in the literature predict variations of the difference Casimir force at the level of 1 pN. An even greater concern is that the model taking into account the dc conductivity of dielectrics violates the Nernst heat theorem [52, 53, 54]. We have reported an apparatus developed at UC Riverside that has the sensitivity of force measurements on the level of 1 fN and is well adapted for the systematic investigation of the proposed effects in a wide range of separations. This apparatus includes an interferometer based atomic force microscope operated in high vacuum over a temperature range from 360 K to 4 K. The proposed experiments are feasible using the developed techniques and will aid in the resolution of theoretical problems on the application of the Lifshitz theory at nonzero temperature to real materials.

Another motivation of the proposed experiments is in the application to nanotechnology. The separations between the adjacent surfaces in micro- and nanoelectromechanical devices are rapidly falling to a region below a micrometer where the Casimir force becomes dominant. Keeping in mind that semiconducting materials are used for micromachines, the detailed investigation of the dependence of the Casimir force on the properties of semiconductors is important. The proposed experiments and related theory clearly demonstrate that it is possible to control the Casimir force with semiconductor surfaces by changing the charge carrier density with doping or excitation. This opens new opportunities discussed above for using the Casimir force in both the operation and function of novel nanomechanical devices.

In addition to the previously performed experiments on the Casimir force (see review [4] and Refs. [5, 6, 7, 8, 21, 23, 24, 25]) currently a number of new experiments have been proposed in the literature. Thus, Ref. [55] proposes to measure the Casimir torque between two parallel birefringent plates with in-plane optical anisotropy separated by either a vacuum or ethanol. In Ref. [56] it is suggested to measure the vacuum torque between corrugated mirrors. References [57, 58, 59] propose the measurements of the Casimir force between metallic surfaces at large separations of a few micrometers as a spherical lens and a plate, a cylinder and a plate or two parallel plates. These experiments are aimed at resolving the theoretical problems arising in the Lifshitz theory when it is applied to real metals. In Refs. [60, 61] a proposal to measure the influence of the Casimir energy on the value of the critical magnetic field in the transition from a superconductor to a normal state has been made. References [62, 63] proposed the measurement of the dynamic Casimir effect resulting in the creation of photons. The experiments proposed here on the difference Casimir force through the use of patterned semiconductor samples or using the insulator-metal phase transition indicate important new promising directions for future investigations in the Casimir effect.

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Figures

Fig. 1. (Color online) Schematic diagram of the experimental setup for the measurement of the difference Casimir force. The patterned Si plate with two sections of different dopant densities is mounted on a piezo below the Au coated sphere attached to a cantilever of an atomic force microscope. The piezo oscillates in the horizontal direction above different regions of the plate causing the flexing of the cantilever in response to the Casimir force.

Fig. 2. (Color online) Steps in the fabrication of the patterned Si plate with patterned doping (see text for more details).

Fig. 3. (Color online) Image of the interferometer based variable temperature atomic force microscope with the force sensitivity up to 1 fN fabricated at UC, Riverside. The critical components are labeled.

Fig. 4. The dielectric permittivity of Si along the imaginary frequency axis for samples with high concentration of charge carriers \tilde{n}_1 and \tilde{n}_2 is shown by the solid lines 1 and 2, respectively. For the sample with a low concentration of charge carriers n the permittivity versus frequency is shown by dashed lines a and b based on whether the static permittivity is finite or infinitely large. The permittivity of Au is indicated by the dotted line.

Fig. 5. The difference Casimir forces versus separation in the case when the higher concentration of charge carriers is equal to \tilde{n}_1 and the sample with a lower concentration, n , is described by a finite or infinitely large static permittivity are shown by the solid line 1a and dashed line 1b, respectively. The analogous difference forces when the higher concentration of charge carriers is equal to \tilde{n}_2 are shown by the solid line 2a and dashed line 2b.

Fig. 6. The difference Casimir pressures versus separation in the case when the higher concentration of charge carriers is equal to \tilde{n}_1 and the sample with a lower concentration, n , is described by a finite or infinitely large static permittivity are shown by the solid line 1a and dashed line 1b, respectively. The analogous difference pressures when the higher concentration of charge carriers is equal to \tilde{n}_2 are shown by the solid line 2a and dashed line 2b.

Fig. 7. Schematic of the experimental setup for the observation of modulation of the Casimir force in an insulator-metal phase transition. Light from a chopped 980 nm laser heats a VO₂ film leading to a phase transition to a state with higher concentration of charge carriers (sapphire substrate is not shown). Cooling in between pulses causes the transition to a state with lower concentration of carriers. The cantilever of an atomic force microscope flexes in response to the difference Casimir force.

Fig. 8. (Color online) Preliminary results on the resistance of VO₂ film grown at UC Riverside as a function

of temperature are shown as black squares (heating) and dots (cooling). **Fig. 9.** (Color online) Morphology of the same VO_2 film, as in Fig. 8, grown by thermal evaporation. The heights of roughness peaks are of about 2 nm. **Fig. 10.** The effective dielectric permittivity of VO_2 film on sapphire substrate along the imaginary frequency axis before and after the phase transition are shown by the solid lines 1 and 2, respectively. The permittivity of Au is indicated by the dotted line. **Fig. 11.** The Casimir force between an Au coated sphere and VO_2 film on sapphire substrate versus separation before and after the phase transition are shown by the solid lines 1 and 2, respectively. **Fig. 12.** The difference of the Casimir forces after and before the phase transition versus separation computed using a finite static dielectric permittivity (solid line) and taking into account the dc conductivity of VO_2 in a dielectric state (dashed line).

Tables

TABLE I: Values of the oscillator resonant frequencies $\omega_{n,i}$, dimensionless relaxation parameters $\Gamma_{n,i}$ and oscillator strengths $s_{n,i}$ of VO₂ film on sapphire substrate before the phase transition.

i	$\omega_{n,i}$ (eV)	$\Gamma_{n,i}$	$s_{n,i}$
1	1.02	0.55	0.79
2	1.30	0.55	0.474
3	1.50	0.50	0.483
4	2.75	0.22	0.536
5	3.49	0.47	1.316
6	3.76	0.38	1.060
7	5.1	0.385	0.99

TABLE II: Values of the oscillator resonant frequencies $\omega_{\tilde{n},i}$, dimensionless relaxation parameters $\Gamma_{\tilde{n},i}$ and oscillator strengths $s_{\tilde{n},i}$ of VO₂ film on sapphire substrate after the phase transition.

i	$\omega_{\tilde{n},i}$ (eV)	$\Gamma_{\tilde{n},i}$	$s_{\tilde{n},i}$
1	0.86	0.95	1.816
2	2.8	0.23	0.972
3	3.48	0.28	1.04
4	4.6	0.34	1.05

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